Estimation Instances

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1. The format of the 9,000,000 numbers is formulated below.

$$y = d_0.d_1d_2d_3d_4d_5d_6 \qquad (1 \le d_0 \le 9).$$

So the range is from 1 to 9.999999.

Because we want to calculate sum of the reciprocals of such 9,000,000 numbers, we can use definite integral method to estimate the value of sum like below.

$$Sum = \sum_{1}^{9.999999} \frac{1}{y}$$

$$Since\ from\ the\ concept\ of\ definite\ integral\ \frac{1}{y}\ from\ 1\ to\ 9.999999, we\ can\ get$$

$$\int_{1}^{9.999999} \frac{1}{y} dy = \lim_{\Delta y \to 0} \sum_{i=1}^{9.999999} \frac{1}{y_i} \times \Delta y = \sum_{i=1}^{9.999999} \frac{1}{y_i} \times \Delta y + o(y)$$

$$Here, o(y)\ is\ Higher\ Order\ Infnitesimal\ of\ y, so\ o(y) is\ a\ really\ tiny\ value\ which\ we\ can\ omit.$$

$$Then, \sum_{i=1}^{9.999999} \frac{1}{y_i} \times \Delta y + o(y) \approx \sum_{i=1}^{9.999999} \frac{1}{y_i} \times \Delta y$$

$$Therefore, we\ can\ get$$

$$\int_{1}^{9.999999} \frac{1}{y} dy \approx \sum_{i=1}^{9.999999} \frac{1}{y_i} \times \Delta y$$

$$Since \int_{1}^{9.999999} \frac{1}{y} dy$$

$$= \ln(9.999999) - ln1 = \ln(9.999999) \approx 2.302584993,$$

$$\sum_{i=1}^{9.999999} \frac{1}{y_i} \times \Delta y \approx 2.302584993$$

$$Here, \Delta y = 0.0000001\ (Interval\ between\ two\ neighboring\ y_i\ )$$

$$Therefore, \qquad Sum = \sum_{i=1}^{9.999999} \frac{1}{y_i} \approx \frac{2.302584993}{\Delta y} = \frac{2.302584993}{0.0000001} = 2302584.993$$

In conclusion, the sum of reciprocals of such 9 million numbers is estimated as 2302584.993.

2. Let’s estimate a sum defined by such formula:

$$Sum = \sum_{y=0}^{2^{23}-1} \frac{1}{1 + y \times 2^{-23}}$$

We can see, it is actually a summation question and the variable (y) in this sum formula is increased with same interval (increment 1 each time.). With such sum question with equal changing intervals, we can always use **definite integral method**

(calculus method) to estimate the sum result. Then we can calculate the definite integral of $\frac{1}{1+i\times 2^{-23}}$ with i range from 0 to $2^{23}-1$.

$$\int_0^{2^{23}-1} \frac{1}{1+y\times 2^{-23}}\,dy = \lim_{\Delta y\to 0} \sum_{y=0}^{2^{23}-1} \frac{1}{1+y\times 2^{-23}} \times \Delta y$$

$$= \sum_{y=0}^{2^{23}-1} \frac{1}{1+y\times 2^{-23}} \times \Delta y + o(y)$$

$$Here, o(y)\ is\ Higher\ Order\ Infnitesimal\ of\ y,$$

$$so\ o(y) is\ a\ really\ tiny\ value\ which\ we\ can\ omit.$$

$$Then, \int_0^{2^{23}-1} \frac{1}{1+y\times 2^{-23}}\,dy = \sum_{y=0}^{2^{23}-1} \frac{1}{1+y\times 2^{-23}} \times \Delta y + o(y)$$

$$\approx \sum_{y=0}^{2^{23}-1} \frac{1}{1+y\times 2^{-23}} \times \Delta y$$

$$Here, \Delta y = 1\ (Interval\ between\ two\ neighboring\ y\ )$$

$$Therefore, \int_0^{2^{23}-1} \frac{1}{1+y\times 2^{-23}}\,dy \approx \sum_{y=0}^{2^{23}-1} \frac{1}{1+y\times 2^{-23}}$$

So, we can directly use the definite integral of $\frac{1}{1+y\times 2^{-23}}$ with y range from 0 to $2^{23}-1$ to estimate the sum we want to calculate.

$$\int_0^{2^{23}-1} \frac{1}{1+y\times 2^{-23}}\,dy = \frac{\ln(1+y\times 2^{-23})}{2^{-23}} \Big|_0^{2^{23}-1}$$

$$= \ln(2-2^{-23}) \times 2^{23} = 5814539.4840225866$$

$$\boldsymbol{Therefore, our\ estimated\ sum\ is\ 5814539.4840225866}$$

In conclusion, if the interval of two successive variables is 1, then we can directly use definite integral method to estimate the value of the sum.